NOTATION

T, temperature; λ, thermal conductivity; c, specific mass heat capacity; ρ, density; $c\gamma = c\rho$, specific volumetric heat capacity; α, heat-transfer coefficient; $2l$, plate thickness. Indices: s, surface; m, medium.

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HEAT TRANSFER BY NATURAL CONVECTION IN SPHERICAL GAS LAYERS

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The experimental data of [1] are correlated in the form of a dimensionless equation which is simple and sufficiently accurate for technical calculations and is applicable in the entire region covered by the experiment — up to $Ra_{dk} = 10^{10}$.

The extensive experimental data [1] on free-convection heat transfer through spherical layers of gas (air, CO_2, H_2) are of independent value and can also provide material for the verification of theoretical solutions in this region. Unfortunately, this valuable experimental material has not yet been analyzed and correlated in an appropriate manner: The correlation carried out in [2] lacks an adequate physical basis and is inconsistent with the main tenets of similarity theory.

An analysis of the conditions of similarity of motion and heat-transfer processes due to natural convection of gas in a region bounded by eccentric spherical boundaries with constant temperatures T_1 and T_2 led to a system of generalized variables for the description of heat transfer on surfaces bounding a spherical layer. In particular, the generalized relation for

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the average heat-transfer characteristic for the surface of the inner sphere was obtained in the form

$$\overline{\mathrm{Nu}}_d = \psi\left(\mathrm{Gr}_d,\ \mathrm{Pr},\ \beta\Delta T,\ \frac{d}{D},\ \eta,\ \frac{T_2}{T_1},\ P_{\text{м}}^*\right). \tag{1}$$

The minimum dimensionless pressure P_M^* in the layer is of direct important in the investigation of the pressure distribution in the considered spherical cavity and in specific conditions is a constant included in the absolute dimensionless pressure P^*. Motion and heat transfer, however, depend mainly on the pressure gradients and, hence, P_M^* can be omitted as a characteristic criterion from (1). Numerical calculations [3, 4] for natural convection in horizontal cylindrical gas layers showed that the ratio T_2/T_1 has no significant effect on the quantitative aspect of heat transfer. Taking into account also that in the case of inertialess free motion, which is the most probable situation in restricted volumes, Gr and Pr are included in the form of the product GrPr = Ra, while the effect of $\beta\Delta T$ is limited mainly to its effect as part of Gr ([5], p. 161), we can put relation (1) in the form

$$\overline{\mathrm{Nu}}_d = f(\mathrm{Ra}_d,\ d/D,\ \eta). \tag{2}$$

For layers formed by concentric spheres,

$$\overline{\mathrm{Nu}}_d = f(\mathrm{Ra}_d,\ d/D). \tag{3}$$

Figure 1 shows the experimental data of [1] for concentric layers, treated in terms of $\overline{\mathrm{Nu}}_{dk}$, Ra_{dk}, and d/D. Figure 1 shows that:

1) the gradient of the straight lines drawn through the experimental points for any value of the geometric similarity criterion d/D is the same and corresponds to the index of the power of Ra_{dk}, which is approximately 0.25 in the whole range covered by the experiment;

2) there is distinct stratification of the data in relation to d/D;

3) in addition, for equal d/D the lines of $\log \overline{\mathrm{Nu}}_{dk} = f(\log \mathrm{Ra}_{dk})$ are stratified according to range of variation of Ra_{dk} and the lines d/D = idem show a characteristic upward trend in the regions of high Ra_{dk}.

The noted features of the experimental data were taken as a basis for their correlation, from which we obtained

$$\overline{\mathrm{Nu}}_{dk} = 0{,}316\left(\frac{d}{D}\right)^{0{,}133} \mathrm{Ra}_{dk}^{0{,}039}\,\mathrm{Ra}_{dk}^{0{,}25}. \tag{4}$$

The subscript k means that the temperature of the surface of the outer sphere (casing) is used as the characteristic temperature.

For practical calculations we can write (4) in the form

$$\overline{\mathrm{Nu}}_{dk} = 0{,}316\left(\frac{d}{D}\right)^{0{,}133} \mathrm{Ra}_{dk}^{0{,}289}. \tag{5}$$

The convective heat transfer across the layer in this case is calculated from the equation

$$Q = \bar{\alpha}(t_1 - t_2)F_1. \tag{6}$$

The limits of applicability of Eq. (5) are given by d/D = 0.118-0.605. For Ra_{dk} the upper limit corresponds to $\mathrm{Ra}_{dk} = 10^{10}$, and the lower limit to the transition from convective heat transfer across the layer to heat transfer by conduction, which in the given case corresponds to the condition

$$\mathrm{Ra}_{dk} = \left[\frac{6{,}33}{(1 - d/D)(d/D)^{0{,}133}}\right]^{3{,}46}. \tag{7}$$

The experimental data are compared with relation (4) in Fig. 2. The deviations of the experimental points from (4), with the exception of three points out of 125, are less than ±15% and can be attributed primarily to the scatter of the initial experimental data (see Fig. 1).

It would be relevant to compare the obtained result with the results of other investigators. In the period 1966-1973 there appeared a series of papers [6-9] dealing with heat

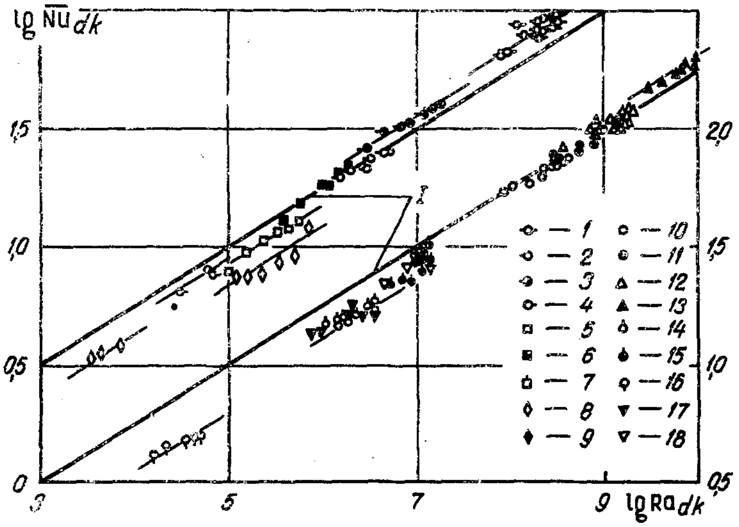

Fig. 1. Experimental data [1] treated in terms of $\overline{Nu}_{dk}$, Ra_{dk}, and d/D: 1) d/D = 0.605, air; 2) 0.6, air; 3) 0.6, CO_2; 4) 0.6, H_2; 5) 0.5, air; 6) 0.5, CO_2; 7) 0.5, H_2; 8) 0.3, air; 9) 0.3, H_2; 10) 0.205, air; 11) 0.205, CO_2; 12) 0.204, air; 13) 0.204, CO_2; 14) 0.195, air; 15) 0.195, CO_2; 16) 0.195, H_2; 17) 0.118, air; 18) 0.118, CO_2; I — straight lines with gradient 0.25.

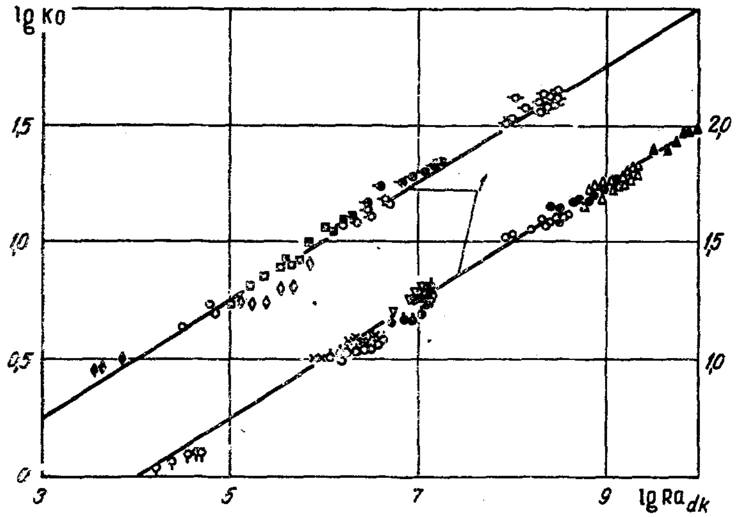

Fig. 2. Comparison of relation (4) with experimental data. 1) Eq. (4); $Ko = \overline{Nu}_{dk}[Ra_{dk}^{0.039}(d/D)^{0.133}]^{-1}$.

transfer by natural convection between two spheres, one of which was contained within the other. The experiments were conducted with air, water, and two silicone fluids in a wide range of Gr and Pr, d/D, and vertical relative eccentricities $e/r_o - r_i$. Papers [6-9] can rightly be regarded as highly inconsistent investigations, in which extensive and reliable experimental data are not correlated correctly enough, and, hence, the accuracy of the obtained correlations is unsatisfactory. Owing to the absence of tables of experimental data in [6, 8, 9] we cannot undertake the correlation of all the experimental data of [1] and [6, 8, 9] on the basis of a relation of type (2). However, we can assess the quality of some of the types of correlation obtained in [9] as a result of investigations [6-9], and compare them with the quality of correlation (4).

For instance, Weber et al. [9], referring to [8], indicate that all the data can be represented by the correlation equation

$$k_{eff}/k = 0.228\,(Ra^*)^{0.226} \quad \text{(a)}$$

For concentric spheres the mean deviation of the data was 15.6% and 76% of the data agreed to within ±20% with the values calculated from this equation; for eccentric spheres the corresponding characteristics were a little lower — 18.2 and 67.6%, respectively. However, Fig. 7 in [9] shows that if we consider all the experimental points, the deviations from the correlation equation for a large number of them reach 60-100% (for instance, in the region of $Ra^* = 2 \cdot 10^3 - 2 \cdot 10^5$). Other features of this correlation are: the radius R_i contained in Ra^* is calculated with the aid of a complex of three formulas: the characteristic temperature used is a certain temperature T_m which is found by means of a complex of two formulas; the convective heat transfer across the layer is calculated from the thermal-conductivity formula for a spherical layer. It is obvious that the quality of such a correlation cannot be regarded as satisfactory owing to the inadequate accuracy and unjustified laboriousness of the determination of k_{eff} and the calculation of Q.

The reasons for this failure are the incorrect construction of the system of generalized variables in form (a). In this case we find that in implicit form $k_{eff} = f(d/D, \ldots)$, and $Ra^* = f(d/D, \eta, \ldots)$, i.e., the independent geometric similarity criteria are included in the characteristic and dependent similarity criteria, which usually leads to difficulties in correlation and less accurate approximation of the experimental data.

This brief analysis of the results of [6-9] merely brings out the advantages of the system of generalized variables in form (2) or (3) and the obtained correlation (5).

NOTATION

$Nu_d = \alpha d/\lambda$, Nusselt number on surface of inner sphere; d, D, diameters; r, r_i, R, r_o, radii of inner and outer spheres; α, heat-transfer coefficient on surface of inner cylinder; [$\alpha \equiv Q_k/(t_1 - t_2)F_1$, $W/m^2 \cdot deg$]; λ, thermal conductivity; F_1, surface of inner sphere; Q, heat transferred by convection across layer; $Ra = g\beta(t_1 - t_2)d^3/\nu^2 \cdot Pr$, Rayleigh number; g, gravitational acceleration; β, coefficient of cubic expansion; ν, kinematic viscosity; Gr, Pr, Grashof and Prandtl numbers; t, °C, T, °K, temperatures; subscripts 1 and 2 attached to t or T refer to the surfaces of the inner and outer spheres, respectively; $t = t_1 - t_2$; $T = T_1 - T_2$; k_{eff}, k, effective and molecular thermal conductivities; $Ra^* = g\beta L^4 \Delta T/\nu a R_i$, modified Rayleigh number; R_i, radius of displaced inner sphere in complex region (for concentric inner sphere $R_i = r_i$); $\eta = e/R - r = e/r_o - r_i$, vertical relative eccentricity; e, vertical eccentricity of inner sphere relative to outer.

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